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Theoretical Aspects of Inventory Optimization: A Mathematical and Statistical Perspective in Formulation the Problems

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Abstract

The pivotal shift from single-use to reusable packaging has recently challenged the concept of packaging ownership. Extant literature have studied supply chain systems using reusable packaging for bundling (known as secondary packaging) or transportation (known as tertiary packaging) of products. Although using reusable packaging for containing products (known as primary packaging) has been tested by more than two dozen of the world's biggest brands (e.g., Nestlé, PepsiCo, and Procter & Gamble), it has not received much attention in studies concerning supply chain systems yet. In this paper, we aim to review the extant literature in light of (1) the environmental and economic costs of reusable packaging, (2) the design of reusable packaging logistics systems, and (3) the implications of operations management for reusable packaging. Based on our analysis of existing studies, we then deliver insights and potential opportunities for future research on reusable packaging.

1.1 Introduction

The pivotal shift from single-use to reusable packaging has recently challenged the concept of packaging ownership. This shift has made a package an asset for the product company, and hence, the company is motivated to make the package as long-lasting and durable as possible. TerraCycle is a small company that has recently compelled more than two dozen of the world's biggest brands such as Nestlé, PepsiCo, and Procter & Gamble to begin testing reusable packaging for their products (Makower, 2019). TerraCycle has unveiled a new circular delivery service for consumers called “Loop”, which is a circular shopping platform that replaces single-use packaging with a durable, reusable one. Consumers can order goods from the Loop website (or that of a partner) and have them delivered like traditional products ordered online. Customers pay a small deposit for a package that has been designed for 100 or more use-cycles. When the container becomes empty, customers place it in a specially designed tote for pickup or, in some cases, can bring it to a retailer. They can choose whether they want that product replenished; if not, their deposit is returned or credited to their account. The empties are sent to a facility where they are washed and refilled. The focus of the Loop's service is on the rotation of primary packages for basic products such as shampoo, toothpaste, ice cream, etc. (the concept of primary packaging will be explained in the next paragraph).

Palsson (2018) classified packages based on their layer or functionality into three different categories: (1) primary packaging which is the packaging that first envelops the product and holds it. This category of packaging is in direct contact with the product; (2) secondary packaging which is an outer packaging layer of the primary packaging and may be used to prevent theft or to bundle primary packages together; and (3) tertiary or transit packaging which is used for bulk handling, warehouse storage, and transport shipping.

Stop Waste and Reusable Pallet and Container Coalition (2007) provided a list of virtues reusable tertiary packaging brings to the system. Reusable transport packages improve workers safety and ergonomics, because (1) their material and design reduce or eliminate injuries due to box cutting, staples, and broken containers, (2) their ergonomically designed handles and access doors improve workers safety, (3) their standardized sizes and weights reduce back injuries, and (4) they reduce the risk of slip and fall injuries by removing in-plant debris. Reusable transport packaging also provides just-in-time delivery of the finished products, because it provides standardized ordering quantities which can improve ordering procedures and inventory tracking. In addition, it provides more frequent shipments of smaller quantities and offers deliveries close to the time of consumption which can reduce the number of days that dollars and inventory are nonproductive.

Reusable secondary packaging can have advantages that are common with the tertiary option (see Stop Waste and Reusable Pallet and Container Coalition, 2007). Both can reduce the product damage, because the risk of packaging failure during the transportation is lower when using reusables compared to when using single-use containers. They can also improve the quality of the finished product delivered to the end user (consumer) as ventilated reusable containers increase shelf-life and freshness. Furthermore, using these packaging systems for shipping products in a supply chain can make substantial cost-savings since cost of reusable packages can be spread over several years. In addition, both packaging systems can be beneficial from waste management perspective as they produce less waste to be managed for recycling or disposal. Finally, one of the main reasons of using such packaging systems is their environmental impacts. By using this type of containers, the need for building disposal facilities or recycling facility centers is dampened. Using this type of containers for delivering products may also reduce the greenhouse gas emission rates and overall energy consumption of the whole system.

Makower (2019) listed three virtues for this category of packaging systems: (1) it moves from disposal or recycling to reuse which is a huge environmental upgrade; (2) it moves from relatively low value packaging materials to arguably luxury or game-changing packaging materials (e.g., from multi-layered plastic film to stainless steel, glass, or engineered plastics); and (3) it brings out new features that could have never been experienced by disposable packages (e.g., a double wall stainless steel container that keeps ice cream frozen for a number of hours after removing it from the freezer).

Different types of reusable packaging are observed with different terms in the literature. For example, “returnable packaging materials” and “returnable transport items” are the terms used for reusable primary and tertiary packaging, respectively (Carrasco-Gallego et al., 2012). Refillable glass bottles for beverages (Goh and Varaprasad, 1986; Del Castillo and Cochran, 1996), gas cylinders (Kelle and Silver, 1989a, Kelle and Silver, 1989b,1989b), containers for chemicals, single-use cameras (Toktay et al., 2000), special packaging designed for transporting medical equipments, wind turbine parts, and steel coils (Rubio et al., 2009) are some examples of returnable packaging materials. Pallets, maritime containers (Crainic et al., 1993), railcars (Young et al., 2002), standardized vessels for fluid transportation, crates, tote boxes, collapsible plastic boxes, trays (Duhaime et al., 2001), roll cages (Carrasco-Gallego and Ponce-Cueto, 2009), barrels, trolleys, pallet collars, racks, lids, etc. Are some examples of returnable transport items being used in business-to-business settings. Returnable transport items can be also used in business-to-customer settings such as supermarket trolleys, baggage trolleys in airports and train stations, and wheeled bins arranged by local councils (Breen, 2006).

Granted the aforementioned virtues for reusable packages, a company willing to adopt such a system for their products should address the following questions before altering their current packaging system: (1) is reusable packaging environmentally and economically feasible? (2) If so, what is the proper design for their logistics system? (3) What are the implications of operations management for reusable packages? In Fig. 1, we illustrate these steps schematically. In this paper, we aim to contribute to the literature by reviewing existing studies in light of the foregoing three questions and identify potential directions/opportunities for future research in this regard. To name a few, the future research could (i) incorporate environmental factors (e.g., carbon taxes, environmental externalities, and eco-costs), consumers’ behavior, and packaging designs in measuring costs, (ii) explore the impact of ownership and third-party logistics in the operations of reusable packaging systems, (iii) analyze such systems under more complicated, and yet realistic, settings (e.g., multiple sender-recipient pairs, variations in the quality of packages, asymmetric information between third parties and senders/recipients), and (iv) consider inter-parties and product-demand-package coordination in managing operations. For a comprehensive discussion regarding these items and many others, one can refer to Section 6.

It should be noted that the reusable primary packaging is a newer concept compared to secondary/tertiary options. Therefore, the existing literature have primarily focused on supply chains using reusable packaging with these options. As a result, we have observed the aforementioned research directions and opportunities with respect to these types of reusable packaging. Nevertheless, given the scope of our proposed research directions (e.g., costs, ownership, complexity of the system, quality of the package, and symmetric information, etc.), all these opportunities could also be applied for a primary reusable packaging option.

To the best of our knowledge, our paper is among the first studies reviewing the operations of reusable packaging systems. Glock (2017) has recently provided a review on returnable transport items, albeit our approach is different from the following standpoints: (1) as one of our classification schemes, we review the literature based on both economical/environmental factors that would impact costs (or criteria to measure these costs); (2) we consider the literature discussing various issues that might arise due to a packaging ownership; (3) we analyze the literature based on various factors in the inventory management of reusable packages, such as a planning horizon, a balance between the supply and demand of packages, and the number of usage for a reusable package; (4) we shed lights on both quantitative and qualitative studies on reusable packaging; and (5) we consider both peer-reviewed journal papers and conference proceedings in searching for relevant studies in the literature

Inventory models deal with the time at which orders for certain goods are to be placed, and the quantity of the order. The research problem concerns ways of optimizing these decisions, taking into account the cost of obtaining the goods, the cost of holding a unit in inventory, and the cost of shortages. More advanced inventory models deal with situations in which there are restrictions on production facilities, storage facilities, time and/or money. For several decades, more OR effort was focused on inventory models than any other area of application.

Changing Role of Inventory management

Inventory management plays a crucial role in the success of any business, impacting various aspects such as cost efficiency, customer satisfaction, and overall profitability. To achieve effective inventory management, a mathematical and statistical approach is essential. This article delves into the world of inventory optimization, exploring how mathematical and statistical techniques can be employed to enhance decision-making and streamline operations.

Major Classes of Operations Research Applications

Over the decades, clusters of problems have proven to be especially amenable to treatment by operations research. They fall into the following six categories of model: inventory, allocation, waiting time, replacement, competitive, and computer simulation.

Inventory Models

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Allocation Models

This class of problems concerns a number of activities to be performed, several alternative methods for carrying them out, and a constraint that the resources or facilities available do not permit each activity to be performed in the most effective way. The analytical challenge of these problems is to combine activities and resources to optimize overall effectiveness. Of particular relevance to this work is the applicability of 'linear programming'—a class of optimization problems that address the interaction of many variables subject to a set of constraints. Subsequent advances in 'nonlinear' and 'dynamic' programming permitted the tackling of more complex assignment problems where a number of the constraints themselves vary with time.

1.1.1 Understanding Inventory Optimization:

Inventory optimization refers to the process of determining the right quantity of items to stock in order to meet customer demand while minimizing costs associated with holding excess inventory. Striking the right balance between having enough inventory to fulfill orders promptly and minimizing carrying costs can be a complex challenge, especially for businesses dealing with a diverse range of products and fluctuating demand patterns.

1.2 Literature review

Multi-item inventory models (Key topic 03)
The perishable inventory models with two- and multi-items (key topic 03) are most frequently found jointly with transport (03–11 in Table B1) and newsvendor (03–18 in Table B1) problems. The remaining papers with multi-item inventory models are (Chakraborty et al., 2013; Das et al., 2014; Duong et al., 2015; Ghosh et al., 2015; Jain et al., 2014; Jiangtao et al., 2013; Purohit and Rathore, 2012; Yadav et al., 2012; Yadavalli et al., 2015; Yan et al., 2013). The number of multi-item inventory models is limited, in comparison to single perishable item models. This imbalance is due mainly to the complexity of multi-item models. Duong et al. (2015) propose a solution for the problem of inventory management in a two-echelon model for perishable and substitutable products with multi-period lifetime. Their model is made more realistic by incorporating multiple inventory characteristics and allowing improvement with performance metrics. Jain et al. (2014) present a study with an inventory system for perishable multi-items having stock dependent demand rates under an inflationary environment of the market. Yadav et al. (2012) develop an inventory model for deteriorating items using a two-warehouse system in a fuzzy environment. Yadavalli et al. (2015) consider a disaster inventory system for two substitutable perishable products. They developed a continuous-review analysis of the disaster inventory.
Perishable goods are modeled in an inventory model with a known fixed lifetime or with a random lifetime. However, a random lifetime with regard of maximum fixed lifetime is also possible, for example Amorim et al. (2013), Avinadav et al. (2014), Chen and Teng (2014), Ketzenberg et al. (2014) and others. Inventory models which only consider lifetime with the known a priori deterministic (fixed) lifetime belong to models for *fixed lifetime*. All other models with probabilistic distributed lifetime (e.g. Weibull), constant, known, unknown deterioration rate etc. are defined as models for *random lifetime* products. We have classified inventory models with simultaneous deteriorating rate and fixed lifetime under models with random lifetime, because the model definition corresponds most fittingly.
Grado and Strauss (1993) and Grado and Strauss (1995) specifically develop inventory models to determine the optimal, minimal cost ordering policy (harvest schedule) of woody biomass for a processing facility. Others consider inventory of raw material at the processing facility (Grunow et al., 2007; Zhang and Hu, 2013; Zhang et al., 2012), final products at the processing facilities (Jones et al., 2003; Miller et al., 1997; Zhang and Hu, 2013) and at distribution channels (Ahumada and Villalobos, 2011a; Ahumada and Villalobos, 2011b) as decision variables. Tsubone et al. (1983, 1984, 1986), on the other hand, develop production planning methods that calculate inventories of semi-processed and final products for a production system comprising of a main processing line (to process raw material into end product to prevent deterioration) and an intermediate processing line (to process the raw material halfway).
The role of market makers as price-setting agents and liquidity providers was initially studied in the inventory models of market microstructure. The inventory-based models consider the trading process as a matching problem in which the market maker or price-setting agent must use prices to balance supply and demand across time (e.g., Garman, 1976; Stoll, 1978; Ho and Stoll, 1981; Cohen et al., 1981). Amihud and Mendelson (1980), and Grossman and Miller (1988) among others examined the impact of inventories on liquidity provision. Based on Grossman and Miller (1988), market liquidity is determined by the demand and supply of immediacy. In their setting, exogenous liquidity events and the risk of delayed trade create a demand for immediacy. Market makers supply immediacy by their presence and willingness to take inventory risk between the arrival of final buyers and sellers. The number of market makers is adjusted in the long run to determine the equilibrium level of liquidity in the market. They showed that the lower is the autocorrelation in rates of return, the higher is the equilibrium level of liquidity. Most inventory models without capital constraints predict that bid-ask spreads are not affected by the market maker's inventory position (e.g., Amihud and Mendelson, 1980; Shen and Starr, 2002). In contrast, O'Hara and Oldfield (1986) showed that spreads depend on inventories if market-makers are risk-averse.

1.3 The Role of Mathematics:

Mathematics provides a robust framework for tackling inventory optimization problems. Various mathematical models and techniques are utilized to address different aspects of inventory management:

1. **Economic Order Quantity (EOQ):** EOQ is a classical inventory management model that aims to find the optimal order quantity that minimizes the total cost of ordering and holding inventory. It takes into account factors such as ordering costs, carrying costs, and demand variability.
2. **Reorder Point (ROP):** ROP is the inventory level at which a new order should be placed to replenish stock before it runs out. It involves considering lead time, demand variability, and desired service level.
3. **Safety Stock:** Safety stock is a buffer inventory maintained to account for uncertainties in demand and lead time. Mathematical formulas, often based on statistical analysis, help determine the appropriate level of safety stock to ensure a desired service level.
4. **Demand Forecasting:** Accurate demand forecasting is crucial for inventory optimization. Mathematical models like time series analysis, moving averages, and exponential smoothing can be used to predict future demand based on historical data.

1.4 Statistical Techniques in Inventory Optimization:

Statistical methods play a significant role in inventory optimization by providing insights into demand patterns, variability, and trends. Here are some key statistical techniques used:

1. **Probability Distributions:** Businesses often experience demand variability. Statistical distributions like the normal distribution or Poisson distribution can model demand patterns, enabling companies to calculate safety stock and reorder points more effectively.
2. **Lead Time Analysis:** Statistical analysis of lead times can help determine the average time it takes for orders to be fulfilled. This information is crucial for calculating reorder points and safety stock levels.
3. **Service Level Analysis:** Service level represents the probability of meeting customer demand. Statistical analysis can assist in setting appropriate service level targets by quantifying the trade-off between customer satisfaction and holding costs.
4. **Monte Carlo Simulation:** This technique involves creating multiple scenarios by randomly sampling from probability distributions. Monte Carlo simulation can be applied to assess the impact of different demand and supply uncertainties on inventory levels and costs.

1.4.1 Integrated Approaches:

Modern inventory optimization often involves integrating mathematical and statistical techniques with advanced technologies such as machine learning and optimization algorithms. These approaches enable businesses to handle complex scenarios and optimize inventory across various product lines, locations, and time horizons.

1.4.2 Challenges and Future Directions:

While mathematical and statistical approaches provide powerful tools for inventory optimization, challenges remain. Dynamic demand patterns, supply chain disruptions, and evolving market conditions require continuous adaptation of models and strategies. Moreover, advancements in data analytics, AI, and optimization algorithms are reshaping the landscape of inventory management, offering more sophisticated solutions for real-time decision-making.

1.5 Conclusion:

Inventory optimization is a critical function for businesses aiming to achieve operational efficiency and customer satisfaction. The combination of mathematical and statistical techniques provides a solid foundation for making informed decisions about order quantities, reorder points, and safety stock levels. By leveraging these tools, businesses can strike the right balance between meeting customer demand and minimizing inventory holding costs, ultimately driving success in a competitive marketplace.

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